

SCALAR-TENSOR σ -COSMOLOGIES

SPIROS COTSAKIS

GEODYSYC, Department of Mathematics, University of the Aegean, 83200, Samos, Greece

E-mail: skot@aegean.gr

JOHN MIRITZIS

Department of Marine Sciences, University of the Aegean, Mitilene 81100, Greece

E-mail: john@env.aegean.gr

We show that the scalar-tensor σ -model action is conformally equivalent to general relativity with a minimally coupled wavemap with a particular target metric. Inflation on the source manifold is then shown to occur in a natural way due both to the arbitrary curvature couplings and the wavemap self-interactions.

1 Scalar-tensor σ -models

Let $(\mathcal{M}^m, g_{\mu\nu})$ be a spacetime (source) manifold, (\mathcal{N}^n, h_{ab}) Riemannian (target) manifold and a \mathcal{C}^∞ map $\phi : \mathcal{M} \rightarrow \mathcal{N}$. We may think of the scalar fields $\phi^a, a = 1, \dots, n$, as coordinates parametrizing the Riemannian target. Our starting point is the general action functional

$$S = \int_{\mathcal{M}} L_\sigma dv_g, \quad dv_g = \sqrt{-g} dx, \quad (1)$$

where

$$L_\sigma = A(\phi)R - B(\phi)Tr_g(\phi^*h) = A(\phi)R - B(\phi)g^{\mu\nu}h_{ab}\partial_\mu\phi^a\partial_\nu\phi^b, \quad (2)$$

where A, B are arbitrary \mathcal{C}^∞ functions of ϕ . We see that S has arbitrary couplings to the curvature and kinetic terms and we call it the scalar-tensor σ -model or, the scalar-tensor wavemap action. Such a theory includes as special cases many of the scalar field models considered in the literature (e.g.,^{1,2,3}). Under compact variations of the families $g_{(s)}$ and $\phi_{(s)}$, $s \in R$ where $\dot{\psi}(s) = [\partial\psi_{(s)}/\partial s]_{s=0}$, the Action Principle, $\dot{S} = 0$, leads to the system,

$$G_{\mu\nu} = \frac{B}{A}h_{ab}\left(\phi^a_{,\mu}\phi^b_{,\nu} - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\phi^a_{,\rho}\phi^b_{,\sigma}\right) \quad (3)$$

$$+ \frac{1}{A}(\nabla_\mu\nabla_\nu A - g_{\mu\nu}\square_g A)$$

$$\square_g\phi^a + \bar{\Gamma}^a_{bc}g^{\mu\nu}\partial_\mu\phi^b\partial_\nu\phi^c + \frac{1}{2}RA^a = 0, \quad \bar{\Gamma} = \Gamma(h) + C, \quad (4)$$

where $A_a = \partial A/\partial\phi^a$, $C^a_{bc} = (1/2)(\delta^a_b C_c + \delta^a_c B_b - h_{bc}B^a)$ and $B_a = \partial \ln B/\partial\phi^a$. Without loss of generality we may perform a conformal transformation on the target metric, $\tilde{h} = Bh$, to find $\bar{\Gamma} = \Gamma(h)$ and so we set from now on $B = 1$ in Eq. (2) and drop the tilde on h . Under a conformal transformation of the source manifold and the target metric redefinition,

$$\tilde{g} = A(\phi)g, \quad \pi_{ab} := \frac{3}{2A^2}A_a A_b + \frac{B}{A}h_{ab} =: Q_a Q_b + J_{ab} \quad (5)$$

(and dropping from the beginning the $\tilde{\Box} \ln \Omega$ term as a total divergence), the original scalar-tensor σ -model action (1)-(2) becomes that of a wavemap minimally coupled to the Einstein term

$$\tilde{S} = \int_{\mathcal{M}} \tilde{L}_\sigma dv_{\tilde{g}}, \quad \tilde{L}_\sigma = \sqrt{\tilde{g}} \left(\tilde{R} - \tilde{g}^{\mu\nu} \pi_{ab} \partial_\mu \phi^a \partial_\nu \phi^b \right). \quad (6)$$

This result shows that all couplings of the wavemap to the curvature are equivalent. Varying this conformally related action, $\dot{\tilde{S}} = 0$, we find the Einstein-wavemap system field equations for the \tilde{g} metric and involving the π_{ab} metric namely,

$$\tilde{G}_{\mu\nu} = \pi_{ab} \left(\phi^a_{,\mu} \phi^b_{,\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{g}^{\rho\sigma} \phi^a_{,\rho} \phi^b_{,\sigma} \right) \quad (7)$$

$$\tilde{\Box}_{\tilde{g}} \phi^a + D_{bc}^a \tilde{g}^{\mu\nu} \partial_\mu \phi^b \partial_\nu \phi^c = 0, \quad D = \Gamma(\mathcal{H}) + T, \quad (8)$$

with $T_{abc} = \partial_c Q_{ab} + \partial_b Q_{ac} - \partial_a Q_{bc}$ and $Q_{ab} = Q_a Q_b$.

2 σ -Inflation

Let us now assume that the source manifold $(\mathcal{M}^m, g_{\mu\nu})$ is the 4-dimensional flat FRW model in the original scalar-tensor wavemap theory (1)-(2). After the conformal transformation (5), the Friedman equation is $H^2 = \frac{1}{3} T_{WM}^{00}$, where the 00-component of the energy-momentum tensor of the wavemap is given by

$$T_{WM}^{00} = \frac{1}{2} \pi_{ab} \dot{\phi}^a \dot{\phi}^b. \quad (9)$$

We see that the time derivative of this may change sign and therefore we find that at the critical points of T_{WM}^{00} the universe inflates,

$$a = a_0 \exp \left(\sqrt{\frac{1}{3} T_{WM,crit}^{00}} t \right). \quad (10)$$

This is the simplest example of a general procedure, which we call σ -inflation, in which inflation is driven both by the coupling $A(\phi)$ and the self-interacting (target manifold is curved!) 'scalar fields' (ϕ^a) which however have no potentials. This mechanism reduces to the so-called hyperextended inflation mechanism¹ when the target space is the real line. On the other hand, when the curvature coupling $A(\phi)$ is equal to one, T_{WM}^{00} can have no critical points since it is always positive and so we have no inflationary solutions. In this case we obtain the so-called tensor-multiscalar models⁴. Inflationary solutions become possible in this case by adding 'by hand' extra potential terms and models of this sort abound.

Details and extensions of the present results will be given elsewhere.

References

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